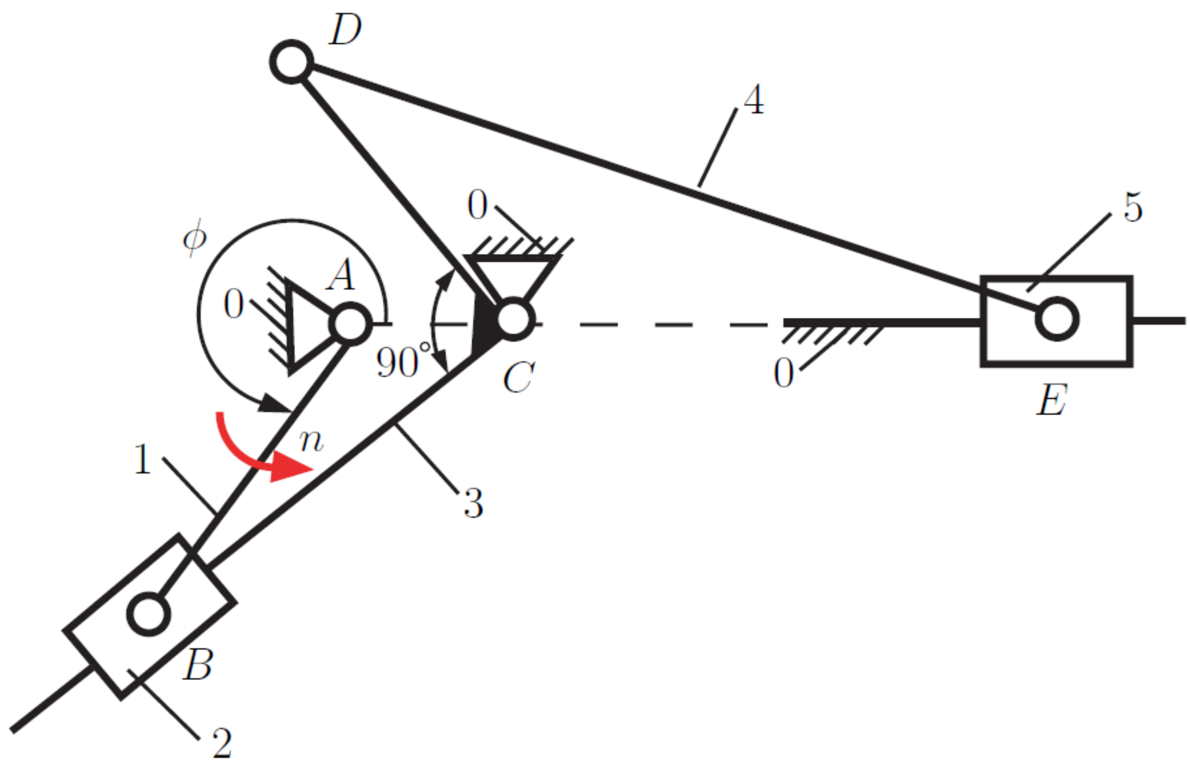
Velocity/Acceleration Analysis

Ryan Brown – Mechanism 9, Index 1

Due 9/10/2015

Mechanism Details



AB [m]	AC [m]	CD [m]	DE [m]	Φ [°]	n [RPM]
0.22	0.08	0.20	0.60	240°	500

From the previous assignment, the following positions will be assumed given:

Positions		
Joint	X	Y
A	0	0
B	-.1100	-.1905
C	.0800	0
D	-.0616	.1412
E	.5215	0

Velocities

1. Velocity analysis for the mechanism using the classical method when the angle of the driver link 1 with the horizontal axis is ϕ.

    1. Point A is fixed, so $\mathbf{v}_A$=$\mathbf{a}_A$=0.

2. The angular velocity of link AB is given in the problem statement in RPM. Converted to Radians per second:

$$\boldsymbol{\omega_{AB}} = n * \frac{2\pi}{60} * \bar{k}$$
$$\boldsymbol{\omega_{AB}} = 500 \text{ RPM} * \frac{2\pi}{60} * \bar{k} \qquad (1)$$
$$\boldsymbol{\omega_{AB}} = 52.359\,\bar{k} \quad \text{rad/sec}$$

3. The angular velocity of link AB is constant, so angular acceleration, $\boldsymbol{\alpha_{AB}}$=0.

4. Point B is on link AB, which is assumed to be rigid. The velocity of point B is:

$$\mathbf{v_B} = \mathbf{v_A} + \mathbf{v_{BA}} = \mathbf{v_A} + \boldsymbol{\omega_{AB}} \text{ x } \mathbf{r_{AB}}$$
$$\mathbf{v_B} = 0 + 52.359\,\bar{k} \text{ x } (-.1100\,\bar{\imath} - .1905\,\bar{\jmath}) \qquad (2)$$
$$\mathbf{v_B} = 9.9744\,\bar{\imath} - 5.7595\,\bar{\jmath} \quad \frac{\text{m}}{\text{s}}$$

The magnitude of the velocity of point B is:

V_B= 11.5178 m/s

5. The position of point C is fixed, so $\mathbf{v_C}$=$\mathbf{a_C}$=0.
6. The direction of the velocity of point B is tangent to the arc created about point A. The direction of the force causing link CB to rotate is perpendicular to link CB. The dot product of the velocity of point B with the vector perpendicular to link CB will give the projected portion of the velocity in the perpendicular direction to link CB; this velocity projection can be used to find the angular velocity of link CB, knowing the positions of C and B.

$$\mathbf{r_{CB}} = \mathbf{r_B} - \mathbf{r_C} = -.19\,\bar{\imath} - .1905\,\bar{\jmath}$$

Slope of $\mathbf{r_{CB}}$ = -.1905/-.91=1.0026

(-) Inverse of slope = -1/1.0026 = -.9974

Vector perpendicular to $\mathbf{r_{CB}}$: $1\bar{\imath} - .9974\,\bar{\jmath}$

Convert to unit vector = $.708\bar{\imath} - .706\,\bar{\jmath}$

Using this vector, take the dot product to find the magnitude of the velocity projection in the same direction:

$$|\mathbf{v_{B/CB}}| = \mathbf{v_B} \cdot (.2047\bar{\imath} - .9788\,\bar{\jmath})$$
$$|\mathbf{v_{B/CB}}| = 11.1292 \quad \frac{\text{m}}{\text{s}} \qquad (4)$$
$$\mathbf{v_{B/CB}} = 7.8795\,\bar{\imath} - 7.8595\,\bar{\jmath} \quad \frac{\text{m}}{\text{s}}$$

Using this velocity, the fixed position of point C, and the vector $\mathbf{r_{CB}}$, the angular velocity of link CB can be found.

$$\mathbf{v_{B/CB}} = \mathbf{v_C} + \mathbf{v_{BC}} = \mathbf{v_A} + \boldsymbol{\omega}_{\mathbf{CB}} \times \mathbf{r_{CB}}$$
$$7.8795\,\bar{\imath} - 7.8595\,\bar{\jmath} = 0 + \boldsymbol{\omega}_{\mathbf{CB}} \times (-.19\,\bar{\imath} - .1905\,\bar{\jmath}) \tag{5}$$
$$\boldsymbol{\omega}_{\mathbf{CB}} = 41.4184\,\bar{k} \quad \frac{\text{rad}}{\text{s}}$$

7. Because CD is fixed to CB, the angular velocity $\boldsymbol{\omega}_{\mathbf{CD}} = \boldsymbol{\omega}_{\mathbf{CB}} = 41.4184\,\bar{k} \quad \frac{\text{rad}}{\text{s}}$
8. The velocity of point D can be found using the angular velocity $\boldsymbol{\omega}_{\mathbf{CD}}$**,** the fixed point C, and the distance CD:

$$\mathbf{r_{CD}} = \mathbf{r_D} - \mathbf{r_C} = (-.0616 - .080)\bar{\imath} + (.1412)\bar{\jmath} = -.1416\bar{\imath} + .1412\bar{\jmath}$$
$$\mathbf{v_D} = \mathbf{v_C} + \mathbf{v_{DC}} = \mathbf{v_C} + \boldsymbol{\omega}_{\mathbf{CD}} \times \mathbf{r_{CD}}$$
$$\mathbf{v_D} = 0 + 41.4184\,\bar{k} \times (-.1416\bar{\imath} + .1412\bar{\jmath}) \tag{6}$$
$$\mathbf{v_D} = -5.8483\,\bar{\imath} - 5.8648\,\bar{\jmath} \quad \frac{\text{m}}{\text{s}}$$

The magnitude of the velocity of point D is:

V_D= 8.2824 m/s

9. The velocity of point E can be found using the velocity of B and velocity of E w.r.t. D:

$$\mathbf{r_{DE}} = \mathbf{r_E} - \mathbf{r_D} = \big(.5215 - (-.0616)\big)\bar{\imath} + (0 - .1412)\bar{\jmath} = .5831\bar{\imath} - .1412\bar{\jmath}$$
$$\mathbf{v_E} = \mathbf{v_D} + \mathbf{v_{ED}} = \mathbf{v_D} + \boldsymbol{\omega}_{\mathbf{DE}} \times \mathbf{r_{DE}}$$
$$\mathbf{v_E} = (-5.8483\,\bar{\imath} - 5.8648\,\bar{\jmath}) + \omega_{DE}\,\bar{k} \times (.5831\bar{\imath} - .1412\bar{\jmath}) \tag{7}$$

The velocity of E is only along the x axis. Simplifying the cross product gives 2 equations and 2 unknowns.

$$v_{D,x} - \omega_{DE} * r_{DE,y} = v_E \tag{8}$$
$$v_{D,x} + \omega_{DE} * r_{DE,x} = 0 \tag{9}$$
$$\boldsymbol{\omega}_{\mathbf{DE}} = 10.0297\bar{k} \ \frac{\text{rad}}{\text{s}}, \quad \mathbf{v_E} = 1.9993\bar{\imath} \ \frac{\text{m}}{\text{s}}$$

The above calculations give the following point velocities, and angular velocities:

Link	Angular Velocity
AB	$52.359\,\bar{k} \ \frac{\text{rad}}{\text{s}}$
CB	$41.4184\,\bar{k} \ \frac{\text{rad}}{\text{s}}$
CD	$41.4184\,\bar{k} \ \frac{\text{rad}}{\text{s}}$
DE	$10.0297\bar{k} \ \frac{\text{rad}}{\text{s}}$

Point	Velocity
A	0
B	$9.9744\,\bar{\imath} - 5.7595\,\bar{\jmath} \ \frac{\text{m}}{\text{s}}$
C	0
D	$-5.8483\,\bar{\imath} - 5.8648\,\bar{\jmath} \ \frac{\text{m}}{\text{s}}$
E	$1.9993\bar{\imath} \ \frac{\text{m}}{\text{s}}$

Accelerations

2. Velocity analysis for the mechanism using the classical method when the angle of the driver link 1 with the horizontal axis is ϕ.
1. The angular velocity of link AB is constant so, $\boldsymbol{\alpha}_{\mathbf{AB}} = 0$.

2. The acceleration of point B is:

$$\mathbf{a_B} = \mathbf{a_A} + \boldsymbol{\alpha}_{\mathbf{AB}} \mathrm{x}\, \mathbf{r_B} + \boldsymbol{\omega}_{\mathrm{AB}} \mathrm{x}\, (\boldsymbol{\omega}_{\mathrm{AB}} \mathrm{x}\, \mathbf{r_B})$$

$$\mathbf{a_B} = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ 0 & 0 & \alpha_{\mathrm{AB}} \\ -.1100 & -.1905 & 0 \end{vmatrix} + (52.359\, \bar{k})\mathrm{x}\left((52.359\, \bar{k})\mathrm{x}\, (-.1100\, \bar{\imath} - .1905\, \bar{\jmath})\right) \tag{10}$$

$$\mathbf{a_B} = 301.5611\, \bar{\imath} - 522.2491\, \bar{\jmath} \quad \frac{\mathrm{m}}{\mathrm{s}^2}$$

3. The normal acceleration of point B is:

$$\mathbf{a_B^n} = \boldsymbol{\omega}_{\mathrm{AB}} \mathrm{x}\, (\boldsymbol{\omega}_{\mathrm{AB}} \mathrm{x}\, \mathbf{r_B}) = 301.5611\, \bar{\imath} - 522.2491\, \bar{\jmath} \quad \frac{\mathrm{m}}{\mathrm{s}^2} \tag{11}$$

4. The normal acceleration is parallel to the vector $\mathbf{r_B}$ and the orientation is toward the center of rotation A (from B to A). The tangential acceleration of point B is:

$$\mathbf{a_B^t} = \boldsymbol{\alpha}_{\mathbf{AB}} \mathrm{x}\, \mathbf{r_B} = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ 0 & 0 & \alpha_{\mathrm{AB}} \\ -.1100 & -.1905 & 0 \end{vmatrix} = 0\, \bar{\imath} - 0\, \bar{\jmath} \quad \frac{\mathrm{m}}{\mathrm{s}^2} \tag{12}$$

5. The tangential acceleration is perpendicular to the vector $\mathbf{r_B}$ and the orientation given by the vector $\boldsymbol{\alpha}_{\mathbf{AB}}$. Point C is fixed, so the acceleration of point C is $\mathbf{a_C} = 0$.

6. The angular acceleration of link BC is found using:

$\mathbf{a_{B/BC}}$ is found with the dot product **$\mathbf{a_{B/BC}}$=($\mathbf{a_B}$·** $(.708\bar{\imath} - .7062\, \bar{\jmath})) * (.708\bar{\imath} - .7062\, \bar{\jmath}) = -109.9574\bar{\imath} + 109.6778\, \bar{\jmath}\, \frac{\mathrm{m}}{\mathrm{s}^2}$

$$\mathbf{a_{B/BC}} = \mathbf{a_C} + \boldsymbol{\alpha}_{\mathbf{CB}} \mathrm{x}\, \mathbf{r_{CB}} + \boldsymbol{\omega}_{\mathbf{CB}} \mathrm{x}\, (\boldsymbol{\omega}_{\mathrm{CB}} \mathrm{x}\, \mathbf{r_{CB}})$$

$$-109.9574\bar{\imath} + 109.6778\bar{\jmath} = \boldsymbol{\alpha}_{\mathbf{CB}} \mathrm{x}(-.19\bar{\imath} - .1905\bar{\jmath}) + (325.9419\bar{\imath} + 326.7997\bar{\jmath}) \tag{13}$$

Solving for $\boldsymbol{\alpha}_{\mathbf{CB}}$, $\boldsymbol{\alpha}_{\mathbf{CB}} = 483.189\, \bar{k} \quad \frac{\mathrm{rad}}{\mathrm{s}^2}$

7. Because link CD is fixed to link CB, the angular acceleration $\boldsymbol{\alpha}_{\mathbf{CD}} = \boldsymbol{\alpha}_{\mathbf{CB}} = 483.189\, \bar{k} \quad \frac{\mathrm{rad}}{\mathrm{s}^2}$

8. The acceleration of point D is found using:

$$\mathbf{a_D} = \mathbf{a_C} + \boldsymbol{\alpha}_{\mathbf{CD}} \mathrm{x}\, \mathbf{r_{CD}} + \boldsymbol{\omega}_{\mathrm{CD}} \mathrm{x}\, (\boldsymbol{\omega}_{\mathrm{CD}} \mathrm{x}\, \mathbf{r_{CD}})$$

$$\mathbf{a_D} = 0 + (483.189\, \bar{k})\mathrm{x}(-.8616\bar{\imath} + .1412\bar{\jmath}) + (58.9273\bar{\imath} - 9.6571\bar{\jmath}) \tag{14}$$

$$\mathbf{a_D} = -9.2990\bar{\imath} - 425.9727\bar{\jmath}\, \frac{\mathrm{m}}{\mathrm{s}^2}$$

9. The acceleration of point E is found using:

$$\mathbf{a_E} = \mathbf{a_D} + \boldsymbol{\alpha}_{\mathbf{DE}} \mathrm{x}\, \mathbf{r_{DE}} + \boldsymbol{\omega}_{\mathrm{DE}} \mathrm{x}\, (\boldsymbol{\omega}_{\mathrm{DE}} \mathrm{x}\, \mathbf{r_{DE}})$$

$$\mathbf{a_E} = (-9.2990\bar{\imath} - 425.9727\bar{\jmath}) + \boldsymbol{\alpha}_{\mathbf{DE}} \mathrm{x}(.5831\bar{\imath} - .1412\bar{\jmath}) + (-2.3385\bar{\imath} + .5663\bar{\jmath}) \tag{15}$$

The acceleration of point E is only in the x direction. Simplifying the cross product gives 2 equations and 2 unknowns.

$$\mathrm{a_{D,x}} - \alpha_{\mathrm{DE}} * \mathrm{r_{DE,y}} = a_{\mathrm{E}} \tag{16}$$

$$\mathrm{a_{D,x}} + \alpha_{\mathrm{DE}} * \mathrm{r_{DE,x}} = 0 \tag{17}$$

$$\boldsymbol{\alpha}_{\mathbf{DE}} = .0627\bar{k} \quad \frac{\mathrm{rad}}{\mathrm{s}^2}, \quad \mathbf{a_E} = -9.3079\bar{\imath} \quad \frac{\mathrm{m}}{\mathrm{s}^2}$$

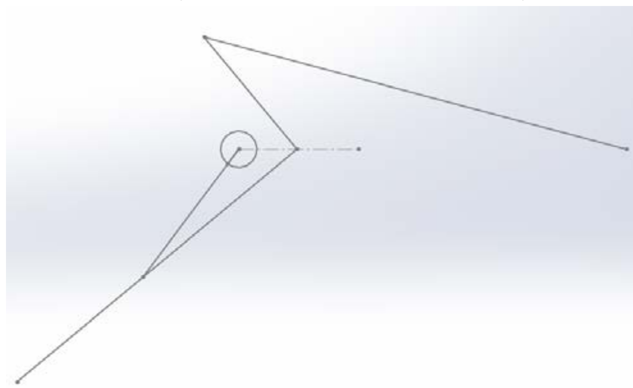
The above calculations give the following point accelerations, and angular accelerations:

Link	Angular Acceleration
AB	0
CB	$483.189\,\bar{k} \quad \frac{\text{rad}}{\text{s}^2}$
CD	$483.189\,\bar{k} \quad \frac{\text{rad}}{\text{s}^2}$
DE	$.0627\bar{k} \quad \frac{\text{rad}}{\text{s}^2}$

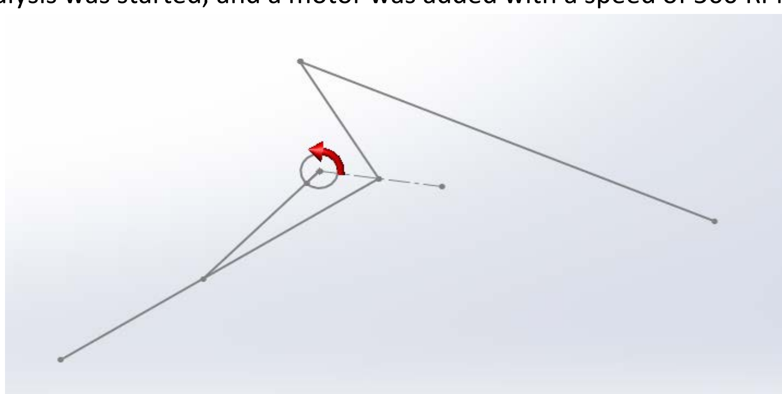
Point	Acceleration
A	0
B	$301.5611\,\bar{\imath} - 522.2491\,\bar{\jmath} \quad \frac{\text{m}}{\text{s}^2}$
C	0
D	$-9.2990\bar{\imath} - 425.9727\bar{\jmath}\ \frac{\text{m}}{\text{s}^2}$
E	$-9.3079\bar{\imath} \quad \frac{\text{m}}{\text{s}^2}$

Method 2: SolidWorks

1. The mechanisms was drawn in “layout” form in a new assembly:



2. A motion analysis was started, and a motor was added with a speed of 500 RPM as shown:



3. The motion analysis method failed and I do not have time to find a different solution.